



Thailand Statistician

Journal of The Thai Statistical Association
(ISSN 1085-4905)

Thailand Statistician
Vol. 7(1) January 2009

<http://www.statassoc.or.th>



Thailand Statistician
Vol. 7(1) January 2009
ISSN 1685-9057

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<http://statassoc.or.th>
Contributed paper

A Single-Level Continuous Sampling Plan for High Quality Production Line

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Received: 30 September 2008

Accepted: 20 January 2009.

Abstract

This paper presents a plan SKIP-CSP-1 for inspection of a high quality continuous production line. The plan is defined by 3 parameters i (the number of consecutive non-defective units that must be produced during a 100% inspection of the line), a fraction f (the specified sampling frequency during a fractional inspection of the line) and k (the number of units for skipping over in an inspection). SKIP-CSP-1 computes 3 performance measures, average fraction inspected (AFI), average outgoing quality (AOQ) and average outgoing quality limit ($AOQL$), for given values of the parameters and incoming fraction of defective units on the line (p). The validity of the performance measure formulas have been tested by extensive simulations. The formulas of performance measures, AFI and AOQ are valid for all the sets of p, i, k, r ($r=1/f$) values. The SKIP-CSP-1 plan has been compared with CSP-1 and CSP-2 plans. On comparing AFI and AOQ , we have found that SKIP-CSP-1 does not give an appreciable difference in the number of units inspected and output quality for low level of p (0.001, 0.003, 0.005) and for all the sets of i, k, r . Further, for higher levels of p , but low levels of k , it also does not give different results. However, compared with CSP-1, we have found that SKIP-CSP-1 gives lower number of units inspected and

output quality than CSP-1, whereas compared with CSP-2, SKIP-CSP-1 gives higher number of units inspected and output quality than CSP-2.

Keywords: continuous sampling plan, high quality production line.

1. Introduction

A continuous sampling plan (CSP) is a plan of sampling inspection for a product consisting of individual units (parts, subassemblies, finished articles etc.) that is manufactured in quantity by an essentially continuous process [1]. A CSP is applicable only to units subject to nondestructive inspection on a GO-NOGO basis. It is intended primarily for use in process inspection of parts, or final inspection of finished articles, where it is desired to have assurance that the percentage of defective units in the accepted product will be less than some prescribed low figure. The original continuous sampling plan (CSP-1) was described by H.F. Dodge and variations of the plan (e.g., CSP-1, CSP-2, CSP-2 CSP-4, CSP-5, CSP-F), have been proposed by many workers.

An inspection procedure in CSP-1 always starts with 100% inspection (screening). This screening is performed until i successive non-defective units are observed. Then the procedure samples only one of the following r units. If the sampled unit is found to be good, then the procedure continues to sample one unit from the next r , etc. As soon as a defective sample unit is observed, the procedure switches back to screening every unit and continues until a further i consecutive good units have been observed. Thus, the inspection procedure consists of alternating periods of 100% inspection and periods with f .100% inspection, where $f = 1/r$ [2].

A CSP-2 differs from a CSP-1 in the sense that during the sampling inspection period the first observed defective unit does not immediately require the procedure to change to 100% inspection. Switching only takes place if another defective is found in the following m sampled units. Frequently, one chooses $m = i$. This choice of m implies that, after discovery of the first nonconforming unit during the f .100% inspection period, the inspection needs to draw only good units in the next i sampled units [3].

Reviews of other CSPs are now available in textbooks (see, e.g., Duncan [4], Grant [5] and Montgomery [6]).

The main objective of this paper is to develop a CSP that can be used for a high quality production line. The paper describes the following:

- 1) The design of a continuous sampling plan for a high quality production line which we call SKIP-CSP-1
- 2) The development of the theory and formulas for important performance measures in SKIP-CSP-1, such as the average fraction inspected (AFI), the average outgoing quality (AOQ) and the average outgoing quality limit ($AOQL$).
- 3) Tests of the validity of the formulas for the performance measures by comparison of the values computed from the formulas with values obtained through extensive simulations.
- 4) A comparison of values of the performance measures of the SKIP-CSP-1 plan with the CSP-1 and CSP-2 plans.

2. Materials and Methods

2.1 The operating procedure of SKIP-CSP-1

Assume that inspection is to be made for only one quality characteristic, so that interest will be centered on one kind of defect. The SKIP-CSP-1 uses 3 parameters for inspection of the units being produced on the production line, namely 2 positive integers i and k , and a fraction f , which are defined by:

- i A number of consecutive non-defective units that must be produced during a 100% inspection of units produced on the line.
- f A sampling frequency for a fractional inspection of units produced on the line ($f=1/r$).
- k A number of units for skipping over in the inspection.

The units sampled during a fractional f inspection of a line must be an unbiased sample of units produced on the line. In all inspection schemes any defective unit that is detected will be replaced immediately by a non-defective unit.

The procedure for inspection of the SKIP-CSP-1 is as follows:

- (1) Inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects. When i units in succession are found clear of defects, discontinue 100% inspection.

- (2) After discontinuing 100% inspection, the selection of the inspection scheme for the new step depends on the results of the preceding step. The selection rules are as follows:
- (2.1) If the number of units inspected consecutively in a 100% inspection of the units is equal to i (no defective units were detected when inspecting 100% of the first i units consecutively), the next k units in succession are skipped over (not inspected) before going on to step 3.
- (2.2) If the number of units inspected consecutively in a 100% inspection of the units is greater than i (at least one defective unit is detected on the line before i units consecutively are found clear of defects), go on to step 3.
- (3) Inspect only a fraction f of the units, selecting individual sample units one at a time from the product flow. This scheme continues until a defective unit is found on the line.
- (4) If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and return to step (1).

In summary, a 100% inspection on the line must continue until the specified number i of consecutive non-defective units are produced on the line. A successful 100% inspection on a line will be followed by an f inspection on the line. In an f inspection, if a defective unit is detected then the plan reverts to a 100% inspection on the line, as in the procedure for inspection of the CSP-1 plan. The different procedure for inspection of SKIP-CSP-1 from CSP-1 is that if a 100% inspection does not find any defects in the first i units inspected then no inspections will be carried out for the next k units.

2.2 The performance measures used in SKIP-CSP-1

The performance measures that we define in SKIP-CSP-1 are generalizations of the performance measures AFI (average fraction inspected), AOQ (average outgoing quality) and $AOQL$ (average outgoing quality limit) used in the conventional CSPs [2]. The measures that we use are as follows:

- The average fraction inspected (AFI).
- The average outgoing quality (AOQ).
- The average outgoing quality limit ($AOQL$).

2.2.1 The average fraction inspected

We have identified 2 phases for the inspection process of the SKIP-CSP-1. Phase 1 consists of a 100% inspection followed by a possible k units with zero inspection. Phase 2 is an f inspection on the line. The average cycle length is the total of the average number of units produced on the line during the inspections of phase 1 and phase 2. The average total inspection is the total of the average number of units inspected on the line during the inspections of phase 1 and phase 2.

We are concerned with the average spacing between defective units on the line. The probability of producing a defective unit is defined to be p . The events of particular interest are a sequence of d non-defective units ($0 \leq d < i$) followed by a defective unit. The complete set of such probabilities for all possible sequences, having respectively $i = 0, 1, 2, \dots, \infty$, defines a probability distribution of random order spacing of defects in uniform product. This is shown in Table 1 in which O represents a non-defective unit, X represents a defective one and $q = 1 - p$. These probabilities are the successive terms in the infinite power series

$$p + pq + pq^2 + pq^3 + \dots = p(1 + q + q^2 + q^3 + \dots) \tag{1}$$

Table 1. The complete set of probabilities for possible sequences that define a probability distribution of random order spacing of defects on the production line.

Sequence	X	OX	OOX	OOOX	...	OOO...OX	...
No. of Term in the Power Series	1	2	3	4	...	i	...
No. of Non-Defective Units before Finding the Next Defect	0	1	2	3	...	$i-1$...
Probability of Occurrence	p	pq	pq^2	pq^3	...	pq^{i-1}	...

The sum of the first i terms is the probability, A , of failing to find the next i units clear of defects, which is

$$A = \sum_{n=0}^{i-1} pq^n = \sum_{n=0}^{i-1} (1-q)q^n = 1 - q^i. \tag{2}$$

In turn, the sum of all terms beyond the i th term is the probability of finding 0 defects in the next i units, which is

$$B = 1 - A = q^i. \quad (3)$$

The average fraction inspected in the long run is defined by:

$$AFI = \frac{ATI}{ACL} \quad (4)$$

where

ATI = The average number of units inspected on the line during the inspections of phase 1 and phase 2 (Average total inspection),

ACL = The average number of units produced on the line during the inspections of phase 1 and phase 2 (Average cycle length).

The average number of units produced on the line during phase 1

We define l as the average number of units produced on the line during phase 1. When inspect 100% of the units consecutively as produced on the line, 2 cases can be happened. For case 1 is the number of units inspected consecutively in a 100% inspection of the units is equal to i (no defective units were detected when inspecting 100% of the first i units consecutively). The next k units in succession are skipped over (not inspected) before inspect only a fraction f of the units. Then $i+k$ of units that will be passed with the probability, B as define in (3). For case 2 is the number of units inspected consecutively in a 100% inspection of the units is greater than i (at least one defective unit is detected on the line before i units consecutively are found clear of defects). We define h as the average number of units inspected in this failure sequence. The average h is

$$h = \frac{p}{1-q^i} (1 + 2q + 3q^2 + 4q^3 + \dots + iq^{i-1})$$

This may be evaluated as follows:

$$\begin{aligned} h &= \frac{p}{1-q^i} \left(\frac{1-q^i(1+pi)}{p^2} \right) \\ &= \frac{1}{p(1-q^i)} [1-q^i(1+pi)]. \end{aligned} \quad (5)$$

The next step is to determine the average number of failure sequences that will be encountered before finding i units clear of defects. This average number designated as G , may be found from the probability distribution of all possible numbers of failure sequences, expressed by the infinite series

$$B(1 + A + A^2 + A^3 + \dots) \quad (6)$$

The successive terms are the probabilities of occurrence of 0, 1, 2, 3, etc. failure sequences before finding i units clear of defects. G is given by the sum of the infinite series

$$G = B(0 + 1A + 2A^2 + 3A^3 + \dots) = BA(1 + 2A + 3A^2 + 4A^3 + \dots)$$

Summing the series, we have $G = BA \frac{1}{(1-A)^2} = \frac{1-q^i}{q^i}$. (7)

The average number of units produced on the line during the inspections is $Gh + i$ units with probability of this case, A as define in (2), and it is therefore

$$\begin{aligned} Gh + i &= \left(\frac{1-q^i}{q^i} \right) \left(\frac{1}{p(1-q^i)} [1-q^i(1+pi)] \right) + i \\ &= \frac{1-q^i}{pq^i} \end{aligned} \quad (8)$$

Now l is the average number of units produced on the line during phase 1. We have

$$\begin{aligned} l &= (i+k)B + (Gh+i)A \\ &= (i+k)(1-p)^i + \frac{(1-(1-p)^i)^2}{p(1-p)^i} \end{aligned} \quad (9)$$

The average number of units produced on the line during phase 2

We define v as the average number of units produced on the line during phase 2 before a defect is found. v will be $1/f$ times the average number of individual sample units inspected in such a period. The average number of sample units inspected in a

period of sampling inspection will thus be the average defect spacing for product having fractional defective, p , and therefore it is given by the infinite series:

$$H = p(1 + 2q + 3q^2 + 4q^3 + \dots)$$

$$\text{Summing the series, we have } H = \frac{p}{(1-q)^2} = \frac{1}{p} \quad (10)$$

$$v = \frac{1}{f} H = \frac{1}{fp} \quad (11)$$

The average cycle length

The average cycle length (*ACL*) is the sum of the average number of units produced on the line during the inspections of phase 1 and phase 2, we have

$$ACL = l + v \quad (12)$$

where l and v as define in (9) and (11) respectively.

The average number of units inspected on the line during phase 1

We define u as the average number of pieces inspected during a 100% inspection followed by a possible k units with zero inspection. We have

$$\begin{aligned} u &= iB + (Gh + i)A \\ &= i(1-p)^i + \frac{(1-(1-p)^i)^2}{p(1-p)^i} \end{aligned} \quad (13)$$

The average number of units inspected on the line during phase 2

For phase 2, the average number of units inspected is fv for fractional sampling inspections.

The average total inspection

The average total inspection (*ATI*) is the sum of the average number of units inspected on the line during the inspections of phase 1 and phase 2, we have

$$ATI = u + fv \quad (14)$$

2.2.2 The average outgoing quality

Suppose a SKIP-CSP-1 is selected by choosing specific values of i , k and r . The average fraction of defective units that appear in the final output of line is called the average outgoing quality (*AOQ*). The *AOQ* is the average fraction of defective units