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Analytical Expression of Average Run Length for Exponential CUSUM

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ABSTRACT

The Cumulative SUM chart (CUSUM) is widely used in a great variety of practical applications such as finance and economics, medicine, engineering, psychology, signal processing, and in other areas. The Average Run Length (ARL) is the most common characteristic used to design CUSUM as well as other control charts (e.g. EWMA, Shiryayev-Roberts). The ARL is usually computed via Markov chain, Monte Carlo simulations or numerical integral equations approaches. In not many cases the solution for the ARL can be found in explicit formula. In this paper we use the integral equation method to derive analytical solutions for the ARL when CUSUM is employed. We derive the ARL for CUSUM chart assuming that the random observations are independent and identically distributed (i.i.d.) exponentially distributed. Checking the accuracy of results, we found an excellent agreement between numerical solutions and the closed form expressions. In addition, the performance comparison of CUSUM and EWMA charts are presented for detecting changes in process mean.

Keywords: Cumulative SUM chart, Average Run Length, Integral Equation.

Mathematics Subject Classification: 62J12, 62G99

Computing Classification System: I.4

1. INTRODUCTION

Control chart is an effective tool in statistical process control for detecting changes in a processes, and uses for measuring, controlling and improving quality in areas such as industrial statistics and manufacturing (see Mason and Antony (2000)), in finance and economics (Golosnoy and Schmid (2006)), computer sciences and telecommunications (Mazalov and Zhuravlev (2002)), epidemiology (Sitter *et al.*(1990)) public health surveillance (Frisen (1992)) and in other areas of applications.

All popular charts such as Shewhart, Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) charts have been developed for detecting changes in a process mean. The traditional Shewhart chart, first introduced by Shewhart (1931), is still widely used in many applications as the main tool for detecting large changes in a process mean. However, the Shewhart chart has been found to be inadequate for detecting small shifts in parameters. In the past few decades, CUSUM and EWMA charts have been proposed as good alternatives to the Shewhart chart for detecting small shifts.

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